NONISOTHERMAL FLOW OF A NONLINEARLY VISCOPLASTIC FLUID IN A RADIAL ANNULAR SLIT

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A theoretical solution is presented to the problem of the nonisothermal flow of a nonlinearly viscoplastic lubricant in a radial annular slit.

Heat transfer from a nonlinearly viscoplastic fluid flowing radially between disks occurs in the flow of lubricants in hydrostatic thrust bearings. Such bearings are used in friction components characterized by a low rotational speed and high unit load.

An important criterion for the optimization of hydrostatic bearings is the energy criterion. Heat release in the bearings should be minimal, since it lessens the accuracy of installation of the bearing, complicates the operation of cooling devices, and increases the probability of deviation from a stable temperature regime.

The well-known solutions for the radial flow of a viscoplastic fluid [1-4] have been obtained for the Shvedovf–Bingham flow equation, which presumes a linear dependence of shear stress on shear rate. At the same time, a number of fluids of practical importance – in particular, lubricants used in hydrostatic bearings – exhibit nonlinearly viscoplastic properties. The solution of the problem of the flow of nonlinearly viscoplastic lubricants described by the Herschel–Balkly model in step bearings was presented in [5].

The authors of [6] proposed a relation to describe the rheological properties of plastic lubricants. This expression closely approximates the experimental data within a broad range of shear rates:

$$\tau = \tau_0 + \eta_0 \exp\left[-(T - \tau_0)/G_0\right] \dot{\gamma}. \tag{1}$$

We will examine the flow of a nonlinearly viscoplastic lubricant in the gap of a hydrostatic thrust bearing. We will ignore rotation of the loaded surfaces, assuming that the bearing as a whole rotates slowly. The flow scheme is shown in Fig. 1. The bearing consists of upper 1 and lower 2 disks. The lower disk has a hole 3 in its central part to admit lubricant, and it rests on a stationary base 4. The external and internal radii of the disks are equal to r_{II} and r_I, respectively, while the gap between the disks is equal to 2h. A load F is applied to the upper disk. Figure 1 also shows a diagram of the shear-stress distribution in the lubricant layer 5 and the velocity profile 6.

To solve the problem, we isolate an annular element of width Δr with the mean radius r_j, as shown in Fig. 1. We assume that the height of the quasisolid region and the pressure gradient are constant within each such element.

Due to the symmetry of the flow, we obtain the solution for the top part of the radial slit.

With allowance for the fact that we are examining the flow of highly viscous non-Newtonian fluids, we can ignore convective heat transfer. We assume that the properties of the lubricant are independent of the temperature and pressure. The equations of motion and energy for the lubricant flow on a section of width Δr within the region of gradient flow ($\bar{h}_0 \le \bar{z} \le 1$) appear as follows in dimensionless form:

$$\frac{\partial \bar{\tau}}{\partial \bar{z}} = -\frac{\partial \bar{p}}{\partial \bar{r}}, \tag{2}$$

$$\frac{\partial^2 \Theta}{\partial \bar{z}^2} + \bar{\tau}\frac{\partial \bar{v}}{\partial \bar{z}} = 0. \tag{3}$$

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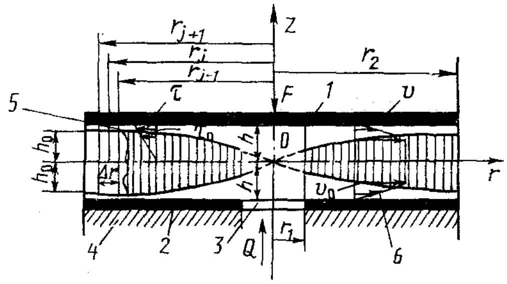

Fig. 1. Lubricant flow in a hydrostatic thrust bearing.

We write the boundary conditions

$$\bar{z}=1;\ \bar{v}=0;\ \Theta=0;\ \bar{z}=\bar{h}_0;\ \frac{\partial\bar{v}}{\partial\bar{z}}=0;\ \frac{\partial\Theta}{\partial\bar{z}}=0. \tag{4}$$

The contact conditions:

$$\bar{z}=\bar{h}_0;\ \bar{v}=\bar{v}_0;\ \Theta=\Theta_0.$$

Since flow velocity is constant in the core and since no energy is dissipated, we write the energy equation for the region of the quasisolid core in the form

$$\frac{\partial^2\Theta}{\partial\bar{z}^2}=0. \tag{5}$$

The boundary conditions:

$$\bar{z}=0;\ \frac{\partial\Theta}{\partial\bar{z}}=0. \tag{6}$$

Proceeding on the basis of the equilibrium equations for the quasisolid core, we express the dimensionless pressure gradient as follows:

$$\frac{\partial\bar{p}}{\partial\bar{r}}=\frac{1}{\bar{h}_0}. \tag{7}$$

Integrating the equations of motion (2) and energy (3) twice with the corresponding boundary conditions (4), with allowance for flow equation (1) and Eq. (7), we obtain the distribution of velocity and temperature in the region of gradient flow on a section of width Δr which contains a quasisolid core of constant height $\bar{h}_0$:

$$\bar{v}=L\{\exp(W)[\bar{h}_0(1+L)-\bar{z}]-\exp(W_1)[\bar{h}_0(1+L)-1]\}, \tag{8}$$

$$\Theta=L^2\left\{\exp(W)\left[L\bar{h}_0^2(1+2L)\left(\frac{\bar{z}}{L\bar{h}_0}-2\right)-\bar{z}^2+2\bar{z}L\bar{h}_0-2L^2\bar{h}_0^2\right]+\right.$$
$$\left.+\exp(W_1)\left[-L\bar{h}_0^2(1+2L)\left(\frac{1}{L\bar{h}_0}-2\right)+1-2L\bar{h}_0+2L^2\bar{h}_0^2\right]+\bar{h}_0(1-2L)(1-\bar{z})\right\}. \tag{9}$$

The velocity of the core $\bar{v}$ is determined from Eq. (8) with $\bar{z}=\bar{h}_0$. Integrating Eq. (5) with allowance for boundary condition (6) and the contact condition, we find that the temperature of the core is constant over the height of the gap and is equal to the temperature Θ_0 determined from Eq. (9) with $\bar{z}=\bar{h}_0$.

The local dimensionless heat flux from the lubricant to the load-bearing surface for an area of width Δr is determined by integrating Eq. (3) with allowance for boundary conditions (4) at $\bar{z}=1$. The final form of the theoretical formula to determine heat flux is

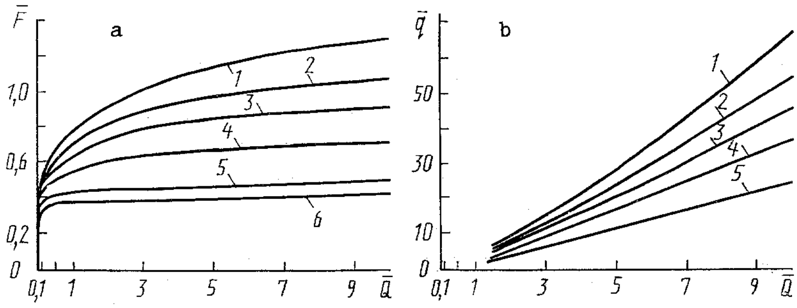

Fig. 2. Dependence of the load-carrying capacity of a bearing (dimensionless) (a) and heat flux to the loaded surface (dimensionless) (b) on lubricant flow rate (dimensionless) for different values of the parameter L: 1) L = 1; 2) 0.7; 3) 0.5; 4) 0.3; 5) 0.1; 6) 0.05.

$$\bar{q} = \left(-\frac{\partial\Theta}{\partial\bar{z}}\right)_{\bar{z}=1} = \frac{L}{\bar{h}_0}\exp(W_1)\,[1 + \bar{h}_0(1+2L)(\bar{h}_0 L - 1)] + L^2\bar{h}_0(1-2L). \tag{10}$$

The mean dimensionless heat flux to the loaded surface of the bearing is determined by summing the amounts of heat transmitted to the surface of each annular element and referring the result to the area of the bearing surface.

To determine the dimensionless height of the core $\bar{h}_0$ for each element of width Δr, we use the continuity equation in integral form:

$$\bar{Q} = \bar{r}\int_0^1 \bar{v}d\bar{z} = \bar{r}\left(\int_{\bar{h}_0}^1 \bar{v}d\bar{z} + \bar{h}_0\bar{v}_0\right). \tag{11}$$

Finally, the expression to determine $\bar{h}_0$ can be represented as

$$\bar{Q} = \bar{r}\left\{L^2\bar{h}_0^2\left[(1-2L) - \exp(W_1)\left(\frac{1}{L\bar{h}_0} - 2\right)\right] + L^2\bar{h}_0^2\exp(W_1)\left(1 - \frac{1}{L\bar{h}_0}\right)\left(1 - \frac{1}{\bar{h}_0}\right)\right\}. \tag{12}$$

Equation (12) is transcendental relative to $\bar{h}_0$, so we used numerical methods to determine the height of the quasisolid core on each section of width Δr.

We used Eq. (7) to determine the dimensionless pressure gradient within each annular element. The load-carrying capacity of the bearing was determined by integrating the linear pressure distribution over the radius within each annular element and then summing the forces acting on each element.

To analyze the effect of the rheological properties of lubricants on the load-carrying capacity of a thrust bearing and the heat flux to the bearing surfaces, we performed calculations on a computer with the use of the formulas obtained above. The results of the calculations are shown in Fig. 2.

In dimensionless coordinates, Fig. 2a shows the dependence of the load-carrying capacity of the bearing on lubricant flow rate with different values of the parameter L for a bearing with a central chamber having a radius $\bar{r}_I = 0.35$.

It is evident from the figure that with an increase in lubricant flow rate, the load-carrying capacity of the bearing increases more rapidly for a large value of the parameter. Given equal flow rates, the highest load-carrying capacity is shown by the bearing in which the lubricant has high values of L (corresponding to an increase in the lubricant's effective viscosity).

In dimensionless coordinates, Fig. 2b shows graphs of the dependence of heat flux to the loaded surfaces of the bearing on lubricant consumption for different values of L.

If we compare the heat flux to the bearing surface for bearings of identical load-carrying capacity but different values of L and, accordingly, different lubricant flow rates, we find from an analysis of Fig. 2 that the heat flux to the loaded surface and the corresponding heat dissipation in the lubricant layer decrease substantially with an increase in the effective viscosity of the lubricant.

TABLE 1. Dependence of Dimensionless Parameters Characterizing the Performance of a Bearing on the Rate of Flow of Lubricant through the Gap with $\bar{r}_1 = 0.35$, $s = 0.0015$, and Different Types of Lubricant

IGP-30 + 5% ceresine-65			IRP-75 + 2.5% ceresine-65		
$\bar{Q}$	$\bar{F}$	$\bar{q}$	$\bar{Q}$	$\bar{F}$	$\bar{q}$
0,5276	3,162	2,852	1,561	6,127	16,59
7,770	6,898	24,35	5,110	12,66	110,9
13,17	9,523	58,21	8,659	16,89	248,5
18,56	11,57	100,38	12,21	20,08	413,9
23,96	13,27	148,77	15,76	22,63	599,6

To confirm these results, we performed numerical calculations for the following lubricant compositions: oil IPG-30 thickened with ceresine-65 in the amount 5% by wt; oil IRP-75 thickened with ceresine-65 in the amount 2.5% by wt.

The rheological constants for these lubricants were determined on a rotation viscometer of the "Reotest" type and were found to be as follows: for oil IGP-30 with 5% (by wt.) ceresine-65: $\tau_0 = 18.0$ N/m^2; $\eta_0 = 0.077384$ N·sec/m^2; $G_0 = 237.0$ N/m^2; for oil IRP-75 with 2.5% (by wt.) ceresine-65: $\tau_0 = 13.0$ N/m^2; $\eta_0 = 0.178212$ N·sec/m^2; $G_0 = 222.1$ N/m^2. Table 1 shows results of numerical calculations of dimensionless parameters characterizing the performance of the bearing.

The results that were obtained fully validate the laws established previously.

The following conclusions can be drawn from the above discussion: in order to reduce lubricant consumption and increase load-carrying capacity when lubricating hydrostatic thrust bearings, a lubricant with the highest possible effective viscosity should be used. In this case, it is possible to regulate the load-carrying capacity of the bearing by changing lubricant consumption within a broad range of values. Such an approach also minimizes dissipative heat losses in the lubricant layer.

NOTATION

τ_0, η_0, G_0, rheological parameters; T, stress intensity; τ and $\dot{\gamma}$, shear stress and rate, respectively; $\bar{\tau} = \tau/\tau_0$, dimensionless shear stress; $L = G_0/\tau_0$, $s = h/r_{II}$, dimensionless parameters; p, lubricant pressure; $\bar{p} = ps/\tau_0$, dimensionless lubricant pressure; $\bar{F} = F/(\tau_0 r_{II}^2 s)$, load-carrying capacity of bearing; h, half the height of the gap between the disks; h_0, height of quasisolid core; $\bar{h}_0$, dimensionless height of quasisolid core; z, axial coordinate; $\bar{z} = z/h$, dimensionless axial coordinate; r, radial coordinate; r_{II} and r_I, external and internal radii of disks; $\bar{r} = r/r_{II}$, dimensionless radial coordinate; $\bar{A}_j = \partial\bar{p}/\partial\bar{r}$, dimensionless pressure gradient on section j; v, lubricant flow velocity; $\bar{v} = \eta_0 v/\tau_0 h$, dimensionless velocity; λ, thermal conductivity of the lubricant; T, lubricant temperature, K; T_0, temperature of the bearing surfaces; $\Theta = (T - T_0)\lambda\eta_0/\tau_0^2h^2$, dimensionless temperature; $\bar{q} = -\partial\Theta/\partial\bar{z}$, dimensionless heat flux; $\bar{v}_0$, Θ_0, dimensionless velocity and temperature of the quasisolid core; Q, volumetric flow rate of lubricant; $\bar{Q} = Q\eta_0/4\pi\tau_0 h^2 r_{II}$, dimensionless flow rate; K, number of annular areas of the subdivision in the calculation of the load-bearing surface of the bearing; $W = (1/L)[(\bar{z}/\bar{h}_0) - 1]$, $W_1 = (1/L)[(1/\bar{h}_0) - 1]$, dimensionless parameters.

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